



**BCH-003-1015003** Seat No. \_\_\_\_\_

**B. Sc. (Sem. V) (CBCS) Examination**

August – 2021

**Mathematics : Paper - 07 (A)**

*(Boolean Algebra & Complex Analysis - I)*

*(New Course)*

**Faculty Code : 003**

**Subject Code : 1015003**

Time :  $2\frac{1}{2}$  Hours]

[Total Marks : 70

**Instruction :**

**1) All the questions are compulsory.**

**2) Attempt any FIVE questions out of TEN.**

**3) Numbers written in the right indicate marks of the questions.**

**1.(A) Answer the following questions briefly. [04]**

- 1) 'R is not reflexive means that R is irreflexive' (TRUE/FALSE) .
- 2) If  $R = \{(2,2), (1,3), (2,3), (3,1)\}$  is a relation on  $A = \{1,2,3\}$  then find  $R^{-1}$ .
- 3) Define: Symmetric Relation.
- 4) Define: Complemented lattice.

**(B) Answer the following question. [02]**

- 1) If  $(L, *, \oplus, ', 0, 1)$  is a bounded lattice then prove that  
(i)  $a * 1 = a$  (ii)  $a \oplus 1 = 1$ .

**(C) Answer the following question. [03]**

- 1) State and Prove Isotonicity Property.

**(D) Answer the following question. [05]**

- 1) Prove that every chain is distributive lattice.

**2.(A) Answer the following questions briefly. [04]**

- 1) Define: Transitive relation.
- 2) Define: Equivalence relation.
- 3) Define: Identity relation.
- 4) Define: Partial order relation.

**(B) Answer the following question. [02]**

- 1) Consider the relation  $R = \{(i, j) / |i - j| = 2\}$  on  $\{1, 2, 3, 4, 5, 6\}$ . Is R is transitive?

- (C) Answer the following question.** [03]
- 1) In the lattice if  $a \leq b$  and  $c \leq d$ , then prove that  $a * c \leq b * d$  and  $a \oplus c \leq b \oplus d$ .
- (D) Answer the following question.** [05]
- 1) Direct product of two lattice is also a lattice.
- 3. (A) Answer the following questions briefly.** [04]
- 1) Define: Sub Boolean algebra.  
 2) Define: Atoms in Boolean Algebra.  
 3) Define: Minterms.  
 4) Define: Maxterms.
- (B) Answer the following question.** [02]
- 1) Draw Hasse Diagram of  $(S_{30}, D)$ .
- (C) Answer the following question.** [03]
- 1) Express the Boolean expression  $a(x_1, x_2, x_3) = x_1 \oplus x_2$  as the sum of product of canonical form.
- (D) Answer the following question.** [05]
- 1) State and Prove De' Morgan's Law.
- 4.(A) Answer the following questions briefly.** [04]
- 1) Define: Boolean isomorphism.  
 2) Sum of all minterms in  $n - variables$  is.....  
 3) The Karnaugh map is useful to minimize the.....  
 4) For POSET( $S_{30}, D$ ) find  $5'$ .
- (B) Answer the following question.** [02]
- 1) In usual notation prove that  $A(x') = A - A(x)$ , Where any  $x \in B$ .
- (C) Answer the following question.** [03]
- 1) Obtain Cube array representation of Boolean function  
 $f(x_1, x_2, x_3, x_4) = x_1(x_2 + x_3x_4)$ .
- (D) Answer the following question.** [05]
- 1) State and Prove Unique Representation Theorem of Boolean algebra.
- 5.(A) Answer the following questions briefly.** [04]
- 1) Define: Analytic function.  
 2) If  $c: |z| = 1$  then  $\int_c \frac{dz}{z-2}$   
 3) Define: Harmonic function.  
 4) Evaluate  $\lim_{z \rightarrow \infty} \frac{2z+3}{z+i}$ .

- (B) Answer the following question.** [02]  
 1) Show that function  $f(z) = z - \bar{z}$  is not an analytic function.
- (C) Answer the following question.** [03]  
 1) is Complex function  $f(z) = z^2$  entire?. Justify your answer.
- (D) Answer the following question** [05]  
 1) Obtain C-R condition in Cartesian form.
- 6.(A) Answer the following questions briefly.** [04]  
 1) If  $f(z) = e^{2z}$  then imaginary part of  $f(z)$  is.....  
 2) Write Cauchy-Riemann equation in Cartesian form.  
 3) Define: Laplace Equation.  
 4) Define: Complex function.
- (B) Answer the following question.** [02]  
 1) Prove that  $f(z) = (3x + y) + i(3y - x)$  is entire function.
- (C) Answer the following question.** [03]  
 1) Find an analytic function  $f(z) = u + iv$  such that,  
 $Im(f'(z)) = 6x(2y - 1)$  and  $f(0) = 3 - 2i$  also find  $f(1 + i)$ .
- (D) Answer the following question.** [05]  
 1) State and Prove Morera's Theorem.
- 7.A) Answer the following questions briefly.** [04]  
 1) Define: Jordan arc.  
 2) Define: limit of complex variable function.  
 3) Define: Smooth curve.  
 4) State Cauchy-Goursat theorem.
- (B) Answer the following question.** [02]  
 1) State fundamental theorem of algebra.
- (C) Answer the following question.** [03]  
 1) State and Prove Cauchy's Inequality.
- (D) Answer the following question.** [05]  
 1) In usual notation prove that  $|\int_C f(z) dz| \leq ML$ .
- 8.(A) Answer the following questions briefly.** [04]  
 1) If  $C : |z - z_0| = r_0 e^{i\theta}$  then  $\int_C \frac{dz}{z - z_0} =$  \_\_\_\_\_.  
 2) If  $L$  is length of Contour  $C$  then  $L =$  \_\_\_\_\_.  
 3) Define: Continuous arc.  
 4) Define: Closed curve.

**(B) Answer the following question.** [02]

1) Find  $\int_0^{2+i} (\bar{z})^2 dz$

**(C) Answer the following question.** [03]

1) In usual notation prove that  $\int_C f(z) dz = ir_0 \int_0^{2\pi} f(z_0 + re^{-i\theta}) d\theta$  where  $C : z - z_0 = r_0 e^{i\theta}$ .

**(D) Answer the following question.** [05]

1) Prove that  $u = r^2 \sin 2\theta$  is Harmonic function and find it's conjugate.

**9.(A) Answer the following questions briefly.** [04]

- 1) The real part of  $f(z) = e^z$
- 2) If  $x + iy = \sqrt{2} + 3i$  then  $x^2 + y = ?$
- 3) Write the formula to find  $f'(z_0)$  for a complex function  $f(z) = u + iv$ .
- 4) Function  $f(z) = \frac{1}{z}$  is analytic function. (TRUE / FALSE)

**(B) Answer the following question.** [02]

1) Prove that  $f'(z) = f(z)$  where  $f(z) = e^z$ .

**(C) Answer the following question.** [03]

1) Evaluate  $\int_C \frac{dz}{(z-1)(z-2)}$ ;  $C: |z| = 3$

**(D) Answer the following question.** [05]

1) Obtain C-R condition for an analytic function  $f(z)$  in Polar form.

**10.(A) Answer the following questions briefly.** [04]

- 1) If  $c: |z - 2| = 5$  then  $\int_C \frac{dz}{z-3} \dots \dots$
- 2) State Green's theorem.
- 3)  $f(z) = \bar{z}$  is not an analytic function.(true/false)
- 4) The real part of  $\frac{2+3i}{3-4i} = \dots \dots$

**(B) Answer the following question.** [02]

1) Show that  $e^{\bar{z}}$  is not analytic.

**(C) Answer the following question.** [03]

1) Evaluate  $\int_C \frac{\sin^6 z}{(z-\frac{\pi}{6})^3} dz$  where  $c: |z| = 2$ .

**(D) Answer the following question.** [05]

1) Find an analytic function  $f(z) = u + iv$  such that  $u - v = x + y$ .